OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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[Turn over

1 Expand $(1-2x)^4$ in ascending powers of x, simplifying the coefficients. [5]

2 **(i)** Find $\int \frac{1}{x^2}\,\mathrm{d}x$. [3]

(ii) The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2}$. Find the equation of the curve, given that it passes through the point $(1, 3)$. [3]

3 **(a)** Express each of the following in terms of $\log_2 x$:

(i) $\log_2(x^2)$, [1]

(ii) $\log_2(8x^2)$. [3]

(b) Given that $y^2 = 27$, find the value of $\log_3 y$. [3]

4 Records are kept of the number of copies of a certain book that are sold each week. In the first week after publication 3000 copies were sold, and in the second week 2400 copies were sold. The publisher forecasts future sales by assuming that the number of copies sold each week will form a geometric progression with first two terms 3000 and 2400. Calculate the publisher's forecasts for

(i) the number of copies that will be sold in the 20th week after publication, [3]

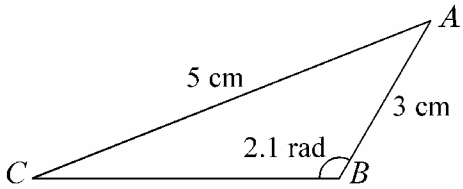
(ii) the total number of copies sold during the first 20 weeks after publication, [2]

(iii) the total number of copies that will ever be sold. [2]

5 **(i)** Show that the equation $15\cos^2\theta° = 13 + \sin\theta°$ may be written as a quadratic equation in $\sin\theta°$. [2]

(ii) Hence solve the equation, giving all values of θ such that $0 \leqslant \theta \leqslant 360$. [6]

6



The diagram shows triangle ABC, in which $AB = 3\,\text{cm}$, $AC = 5\,\text{cm}$ and angle $ABC = 2.1$ radians. Calculate

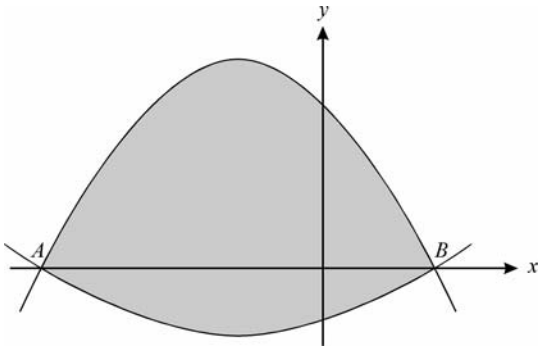
(i) angle ACB, giving your answer in radians, [2]

(ii) the area of the triangle. [3]

An arc of a circle with centre A and radius 3 cm is drawn, cutting AC at the point D.

(iii) Calculate the perimeter and the area of the sector ABD. [4]

7



The diagram shows the curves $y = -3x^2 - 9x + 30$ and $y = x^2 + 3x - 10$.

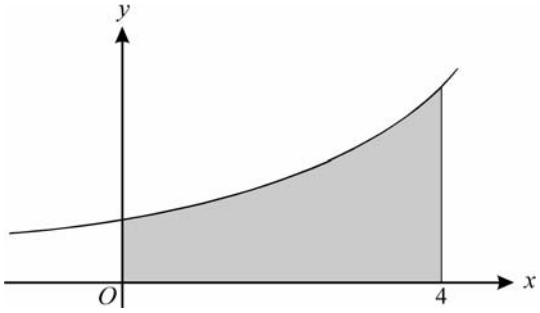
(i) Verify that the curves intersect at the points $A(-5, 0)$ and $B(2, 0)$. [2]

(ii) Show that the area of the shaded region between the curves is given by $\int_{-5}^{2} (-4x^2 - 12x + 40)\,\mathrm{d}x$. [2]

(iii) Hence or otherwise show that the area of the shaded region between the curves is $228\frac{2}{3}$. [5]

[Turn over

8



The diagram shows the curve $y = 1.25^x$.

(i) A point on the curve has *y*-coordinate 2. Calculate its *x*-coordinate. [3]

(ii) Use the trapezium rule with 4 intervals to estimate the area of the shaded region, bounded by the curve, the axes, and the line $x = 4$. [4]

(iii) State, with a reason, whether the estimate found in part **(ii)** is an overestimate or an underestimate. [2]

(iv) Explain briefly how the trapezium rule could be used to find a more accurate estimate of the area of the shaded region. [1]

9 The cubic polynomial $x^3 + ax^2 + bx - 6$ is denoted by $\mathrm{f}(x)$.

(i) The remainder when $\mathrm{f}(x)$ is divided by $(x-2)$ is equal to the remainder when $\mathrm{f}(x)$ is divided by $(x+2)$. Show that $b = -4$. [3]

(ii) Given also that $(x-1)$ is a factor of $\mathrm{f}(x)$, find the value of *a*. [2]

(iii) With these values of *a* and *b*, express $\mathrm{f}(x)$ as a product of a linear factor and a quadratic factor. [3]

(iv) Hence determine the number of real roots of the equation $\mathrm{f}(x) = 0$, explaining your reasoning. [3]

1	$1-8x+24x^2-32x^3+16x^4$	B1 M1 M1 A1 A1	**5** **5**	For first two terms $1-8x$ For expansion in powers of $(-2x)$ For any correct use of binomial coefficients For any one further term correct For completely correct expansion
2	**(i)** $\int x^{-2}\,\mathrm{d}x = -x^{-1}+c$	M1 A1 B1	**3**	For any attempt to integrate x^{-2} For correct expression $-x^{-1}$ (in any form) For adding an arbitrary constant
	(ii) $y=-x^{-1}+c$ passes through $(1, 3)$, so $3=-1+c \Rightarrow c=4$	M1 A1√		For attempt to use $(1, 3)$ to evaluate c For correct value from their equation
	Hence curve is $y=-\frac{1}{x}+4$	A1	**3** **6**	For correct equation
3	**(a)** **(i)** $2\log_2 x$	B1	**1**	For correct answer
	(ii) $\log_2(8x^2)=\log_2 8+\log_2 x^2$	M1		For relevant sum of logarithms
		M1		For relevant use of $8=2^3$
	$=3+2\log_2 x$	A1	**3**	For correct simplified answer
	(b) $2\log_3 y=\log_3 27$	M1		For taking logs of both sides of the equation
	Hence $\log_3 y=\frac{3}{2}$	A1		For any correct expression for $\log_3 y$
		A1	**3** **7**	For correct simplified answer
4	**(i)** $r=\frac{2400}{3000}=0.8$	B1		For the correct value of r
	Forecast for week 20 is $3000\times0.8^{19}\approx43$	M1		For correct use of ar^{n-1}
		A1	**3**	For correct (integer) answer
	(ii) $\frac{3000(1-0.8^{20})}{1-0.8}=14\,827$	M1		For correct use of $\frac{a(1-r^n)}{1-r}$
		A1	**2**	For correct answer (3sf is acceptable)
	(iii) $\frac{3000}{1-0.8}=15\,000$	M1		For correct use of $\frac{a}{1-r}$
		A1	**2** **7**	For correct answer
5	**(i)** LHS is $15(1-\sin^2\theta°)$	M1		For using the relevant trig identity
	Hence equation is $15\sin^2\theta°+\sin\theta°-2=0$	A1	**2**	For correct 3-term quadratic
	(ii) $(5\sin\theta°+2)(3\sin\theta°-1)=0$	M1		For factorising, or other solution method
	Hence $\sin\theta°=-\frac{2}{5}$ or $\frac{1}{3}$	A1		For both correct values
	So $\theta=19.5, 160.5, 203.6, 336.4$	M1		For any relevant inverse sine operation
		A1		For any one correct value
		A1√		For corresponding second value
		A1√	**6** **8**	For both remaining values

6	**(i)**	$\frac{3}{\sin C} = \frac{5}{\sin 2.1} \Rightarrow \sin C = \frac{3}{5}\sin 2.1$	M1		For any correct initial statement of the sine rule, together with an attempt to find $\sin C$
		Hence $C = 0.544$	A1	**2**	For correct value
	(ii)	Angle A is $\pi - 2.1 - 0.5444 = 0.4972$	M1		For calculation of angle A
		Area is $\frac{1}{2}\times 5\times 3\times \sin 0.4972$	M1		For any complete method for the area
		i.e. 3.58 cm^2	A1√	**3**	For correct value, following their C
	(iii)	Sector perimeter is $6 + 3\times 0.4972$	M1		For using $r\theta$ with their A in radians
		i.e. 7.49 cm	A1t		For correct value, following their A
		Sector area is $\frac{1}{2}\times 3^2 \times 0.4972$	M1		For using $\frac{1}{2}r^2\theta$ with their A in radians
		i.e. 2.24 cm^2	A1√	**4**	For correct value, following their A
				9	
7	**(i)**	$-75 + 45 + 30 = 0,\ 25 - 15 - 10 = 0$	B1		For checking one point in both equations
		$-12 - 18 + 30 = 0,\ 4 + 6 - 10 = 0$	B1	**2**	For checking the other point in both
	(ii)	Area is $\int_{-5}^{2}\{(-3x^2 - 9x + 30) - (x^2 + 3x - 10)\}\,\text{d}x$	M1		For use of $\int (y_1 - y_2)\,\text{d}x$
		i.e. $\int_{-5}^{2}(-4x^2 - 12x + 40)\,\text{d}x$, as required	A1	**2**	For showing given answer correctly
	(iii)	*EITHER*: Area is $\left[-\frac{4}{3}x^3 - 6x^2 + 40x\right]_{-5}^{2}$	M1		For integration attempt with one term OK
			A1		For at least two terms correct
			A1		For completely correct indefinite integral
		$= (-\frac{32}{3} - 24 + 80) - (\frac{500}{3} - 150 - 200)$	M1		For correct use of limits
		$= 228\frac{2}{3}$	A1		For showing given answer correctly
		OR: Area under top curve is	M1		For complete evaluation attempt
			A1		For correct indefinite integration (allow for other curve if not earned here)
		$\left[-x^3 - \frac{9}{2}x^2 + 30x\right]_{-5}^{2} = 171\frac{1}{2}$	A1		For correct value
		Area above lower curve is			
		$-\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x\right]_{-5}^{2} = 57\frac{1}{6}$	M1		For evaluation and sign change
		So area between is $171\frac{1}{2} + 57\frac{1}{6} = 228\frac{2}{3}$	A1	**5**	For showing given answer correctly
				9	
8	**(i)**	$1.25^x = 2 \Rightarrow x\log 1.25 = \log 2$	B1		For correct initial use of logs
		Hence $x = \dfrac{\log 2}{\log 1.25} = 3.11$	M1		For correct log expression for x
			A1	**3**	For correct numerical value
	(ii)	$\frac{1}{2}\{1.25^0 + 2(1.25^1 + 1.25^2 + 1.25^3) + 1.25^4\}$	B1		For correct recognition of $h = 1$
			M1		For any use of values 1.25^x for $x = 0, \ldots, 4$
			M1		For use of correct formula
		Area is 6.49	A1	**4**	For correct answer
	(iii)	The trapezia used in **(ii)** extend above the curve	M1		For stating or sketching trapezia above curve
		Hence the trapezium rule overestimates the area	A1	**2**	For stating overestimate with correct reason
	(iv)	Use more trapezia, with a smaller value of h	B1	**1**	For stating that more trapezia should be used
				10	

[Turn over

9	**(i)** $8+4a+2b-6=-8+4a-2b-6$	M1		For equating f(2) and f(−2)
		A1		For correct equation
	Hence $4b=-16\Rightarrow b=-4$	A1	**3**	For showing given answer correctly
	(ii) $1+a-4-6=0$	M1		For equating f(1) to 0 (*not* f(−1))
	Hence $a=9$	A1	**2**	For correct value
	(iii) $f(x)=(x-1)(x^2+10x+6)$	M1		For quadratic factor with x^2 and/or +6 OK
		A1		For trinomial with both these terms correct
		A1	**3**	For completely correct factorisation
	(iv) The discriminant of the quadratic is 76	M1		For evaluating the discriminant
	Hence there are 3 real roots altogether	M1		For using positive discriminant to deduce that there are 2 roots from the quadratic factor
		A1	**3**	For completely correct explanation of 3 roots
			11	